

## PHYS 4203 Physical Applications of Fourier Analysis

**Lectures**: Tuesday 11:35 - 12:55 } (CRN 34626) Via cuLearn, Zoom, and Big Blue Button.

Thursday 11:35 - 12:55

Lecturer: Paul Johns E-mail: Paul.Johns@carleton.ca

Office Hours: Via Big Blue Button in cuLearn at standard times, or by appt.

**Instruction modality:** The lectures are delivered live over Zoom or Big Blue Button, with recordings posted subsequently

on cuLearn. Office hours will be held via Big Blue Button. Information such as announcements and problem assignments will be posted in cuLearn. Your term marks will be available in cuLearn

and you should verify the numbers.

There is one combined cuLearn site for Phys 4203 + Phys 5313.

Calendar Description: Fourier transform, convolution. Sampling theorem. Applications to imaging: descriptors of

spatial resolution, filtering. Correlation, noise power. Discrete Fourier transform, FFT. Filtering of noisy signals. Image reconstruction in computed tomography and magnetic resonance. Laplace

transform. Integral transforms, application to boundary value problems.

**Prerequisite**: Math 3705, or permission of the Physics Department.

Text: R. N. Bracewell, The Fourier Transform and its Applications, McGraw-Hill, either 2nd edition

(1978, ISBN = 0-07-007013-X), the 2nd edition revised (1986)(ISBN = 0-07-007015-6), or the 3rd edition (2000)(ISBN = 0-07-303938-1). [The book is out of print. It's best to get a used copy.

One can also download a pdf from the internet.]

**Other references**: E.O. Brigham, The Fast Fourier Transform, Prentice-Hall, 1974.

E. Butkov, Mathematical Physics, Addison-Wesley, 1968.

H.H. Barrett and W. Swindell, Radiological Imaging, Academic Press, 1981.

W.K. Pratt, Digital Image Processing, 2nd edition, Wiley, 1991.

R.V. Churchill, Fourier Series and Boundary Value Problems, McGraw-Hill, 1941.

**Course content:** Fourier transform: basic characteristics for odd and even functions

Convolution and correlation

Transforms of particular functions

Analysis of linear systems

Filtering

Sampling theorem, aliasing

Fourier series

Discrete Fourier transform Fast Fourier transform (FFT)

Noise Power Spectrum Filtering of noisy data

Relationship of Fourier series to Fourier integral

Two-dimensional Fourier transform Resolution descriptors in imaging

Image reconstruction from projections - computed tomography (CT)

MR reconstruction with phase encoding

Fraunhofer diffraction

Laplace transform, with application to circuit analysis

Other transforms - e.g. Hadamard, wavelet

Applications of integral transforms to boundary value problems

(as time allows)

**Learning objectives:** 

Upon completion of this course,

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- 1. students will have mastery of the mathematics of convolution, Fourier analysis, and their application to linear systems in one dimension and in two and three dimensions.
- 2. students will visualize signal analysis problems in both the normal domain (time or space) and the reciprocal domain (time frequency or spatial frequency).
- 3. at a senior undergraduate level, students will be able to define and analyse multistep analytic problems in signal analysis, including in imaging, using Fourier transform tools and using tools from prior courses in calculus and algebra, and will be able to document their analysis for others.
- 4. students will have an introductory understanding of the Discrete Fourier Transform and of algorithms for its calculation.
- 5. at an introductory level, students will understand and be able to apply the mathematical bases of image reconstruction in CT and MRI.
- 6. students will have a sense of the history of the field, having been introduced to Joseph Fourier and to other mathematicians and physicists including Abel, Nyquist, Heaviside, Shannon, Hounsfield, and Cormack.

## **Problem assignments:**

Expect about 7 problem sets, one every third lecture.

Late assignments will not be accepted.

Clarity, rigour, and organization are important parts of your solutions.

Test & exam:

- 1. There will be two tests, each with both a written and an oral component. The written component will be held during the lecture period. The likely dates are Thurs Oct 8 and Thurs Nov 5. The oral component will be scheduled outside class hours. You may use your notes, class handouts previously downloaded from cuLearn, and the text (Bracewell). No other aids.
- 2. There will be a 3.0 hour online exam in which you may use your notes, class handouts previously downloaded from cuLearn, and the text (Bracewell). No other aids.

**Grading scheme:** 

The final marks for the course will be calculated as, Assignments = 45%

Tests (2) = 30% Exam = 25%

**Course schedule:** 

The first class will be Thursday September 10 at 11:35, and the final lecture will be Thurs Dec 10.

Deferred final exam:

This will replace only the Exam portion of the course mark, 25%. Hence students who earn  $\lesssim 28$  out of the possible 75 marks for the term need to be aware that it is likely pointless to attempt a deferred exam. Deferred Exams for the 2020 Fall term will be 2021 January 22-24 and 29-31. For more information see carleton.ca/registrar/special-requests/deferral/.

Privacy and recordings:

The lectures will be recorded and posted to cuLearn. If you do not wish to be identified during recordings, e.g. when asking questions, please consult with Prof. Johns. Options such as private chat are available.

Copyright:

Classroom teaching and learning activities, including lectures, discussions, presentations, etc., by the instructor and by students, are copy protected and remain the intellectual property of their respective author(s). All Phys 4203 and Phys 5313 course materials, including PowerPoint and pdf files and other materials, are also protected by copyright and remain the intellectual property of their respective author(s).

Students registered in the course may take notes and make copies of course materials for their own educational use only. You may not allow others to reproduce or distribute course materials. You may not make your own recordings of lectures or office hours. Students are not permitted to reproduce or distribute course materials publicly for commercial or non-commercial purposes.

**Copying, plagiarism and other forms of cheating**: The attention of all students is drawn to Section 10.1 of the 2020 Academic Regulations of the University:

calendar. carleton. ca/undergrad/regulations/academic regulations of the university/academic-integrity-and-offenses-of-conduct/#academic-integrity-policy. The conduct of the conduct of

Working through problems is essential in developing a deep understanding of Fourier analysis. Students are permitted to discuss concepts and strategies related to solving the homework assignments. The work handed in, however, must be your own. Submitting an examination or test of any kind, or an assignment, that is copied in whole or in part from someone else is considered plagiarism, which is an academic misconduct offence. This includes copying the full solution or any part of the solution from an online resource like Chegg, solutions manuals, examples posted at Carleton or elsewhere, or from any other type of unauthorized source.

**Use of e-Proctoring system**: This course has timed tests and final examination. The Carleton University e-Proctoring system may be used for these, and requires the use of webcams, microphones, and smart phones.

**Academic Accommodation** – You may need special arrangements to meet your academic obligations during the term. For an accommodation request the processes are as follows. See also <u>students.carleton.ca/course-outline</u>.

Pregnancy obligation – Please contact Prof. Johns with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. For more details, visit the Equity Services website: carleton.ca/equity/wp-content/uploads/Student-Guide-to-Academic-Accommodation.pdf

Religious obligation – Please contact Prof. Johns with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. For more details, visit the Equity Services website: <a href="mailto:carleton.ca/equity/wp-content/uploads/Student-Guide-to-Academic-Accommodation.pdf">carleton.ca/equity/wp-content/uploads/Student-Guide-to-Academic-Accommodation.pdf</a>

Academic accommodations for students with disabilities – If you have a documented disability requiring academic accommodations in this course, please contact the Paul Menton Centre for Students with Disabilities (PMC) at 613-520-6608 or email <a href="mailto:pmc@carleton.ca">pmc@carleton.ca</a> for a formal evaluation, or contact your PMC coordinator to send Prof. Johns your Letter of Accommodation at the beginning of the term. You must also contact the PMC no later than two weeks before the first in-class scheduled test or exam requiring accommodation (if applicable). After requesting accommodation from the PMC, meet with Prof. Johns as soon as possible to ensure accommodation arrangements are made. See <a href="mailto:carleton.ca/pmc">carleton.ca/pmc</a>. Requests for accommodation for the exam must be made by 2020 Nov 13 as per <a href="mailto:carleton.ca/registrat/registration/dates-and-deadlines/">carleton.ca/registrat/registration/dates-and-deadlines/</a>.

Survivors of sexual violence – As a community, Carleton University is committed to maintaining a positive learning, working and living environment where sexual violence will not be tolerated, and where survivors are supported through academic accommodations as per Carleton's Sexual Violence Policy. For more information about the services available at the university and to obtain information about sexual violence and/or support, visit <a href="mailto:carleton.ca/sexual-violence-support">carleton.ca/sexual-violence-support</a>.

Accommodation for student activities — Carleton University recognizes the substantial benefits, both to the individual student and for the university, that result from a student participating in activities beyond the classroom experience. Reasonable accommodation must be provided to students who compete or perform at the national or international level. Please contact Prof. Johns with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. See <a href="mailto:carleton.ca/senate/wp-content/uploads/Accommodation-for-Student-Activities-1.pdf">carleton.ca/senate/wp-content/uploads/Accommodation-for-Student-Activities-1.pdf</a>.